CIVIL ENVIRONMENTAL AND GEOMATIC ENGINEERING

MEng E&AD

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Mechanics of Buildings – Lab Test - Beams

Basic information for the analysis:

- Two different restraint conditions are considered: propped cantilever (*Figure 1*) and fixed ends (*Figure 2*).
- The yielding strength of the steel is f_{yk} = 282 MPa.
- The span's length is L = 750 mm.
- The cross section has base equal to *7.9 mm* and height equal to *7.9 mm (i.e., 7.9x7.9 mm)*.
- A point load P is applied at midspan.

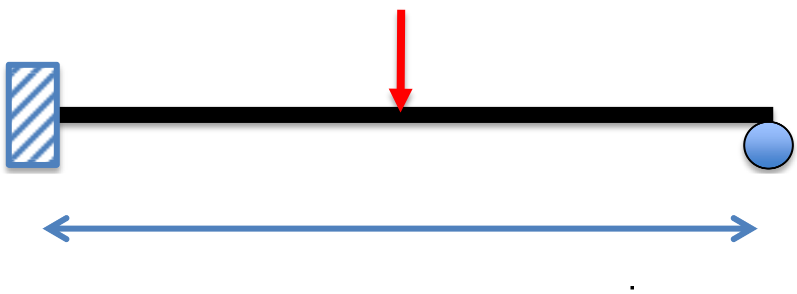

Figure 1: Propped Cantilver Beam

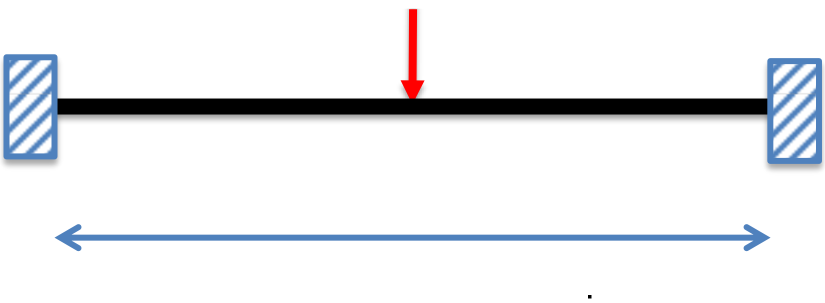

Figure 2 Fixed Ends Beam

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PRE – EXPERIMENT CALCULATIONS

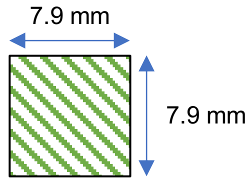

Figure 3 Cross-Section of Beam

1. We have the cross section of the beam with
   b = 7.9 mm
   h = 7.9 mm

   Elastic Section Modulus, Z

   $Z = bh^2/6$

   ⇨ $Z = 82.17\ mm^3 = 8.217 \times 10^{-8}\ m^3$

   Plastic Section Modulus, Z_p

   $Z_p = bh^2/4$

   ⇨ $Z_p = 123.26\ mm^3 = 1.2326 \times 10^{-7}\ m^3$

   We have been given Yielding Strength, $f_{yk} = 282\ MPa = 282 \times 10^6\ Pa$

   Yielding Moment, M_y

   $M_y = f_{yk} .(Z)$

   ⇨ $M_y = 23.172\ Nm$

   Plastic Moment, M_p

   $M_p = f_{yk} .(Z_p)$

   ⇨ $M_p = 34.758\ Nm$

   Shape Factor, ν

   $\nu = Z_p / Z$

   ⇨ $\nu = 6/4$

   ⇨ $\nu = 1.5$

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2. Condition 1: Propped Cantilever

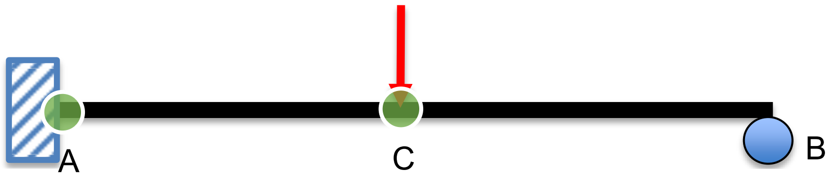

Figure 4 Plastic Hinges on Propped Cantilever

Number of Plastic Hinges = 2
One hinge located at midspan (C), and one Hinge located at the fixed support (A).

Condition 2: Fixed Ends

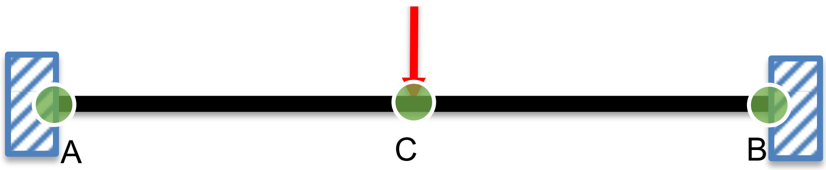

Figure 5 Plastic Hinges on Fixed End Beam

The beam has 3 degrees of indeterminancy.
However, due to horizontal equilibrium equations
$H_A = H_B = 0$
Thus, degree of redundancy is reduced to r = 2

Number of Plastic Hinges = r +1 = 3

One hinge will be formed at midspan (C) and two hinges at either end at fixed supports (A,B).

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3. <u>Condition 1: Propped Cantilever</u>

In the elastic range we have the bending moments:

$M_A = -3PL/16$, and $M_C = 5PL/32$
⇨ $P_y = (32/5L) \cdot M_y$
⇨ $P_y = (32/3.75) \cdot (23.172)$
⇨ $P_y = 197.73$ N

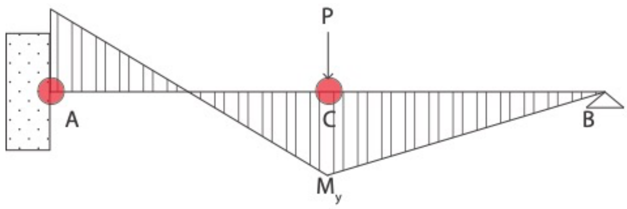


Figure 6 Propped Cantilever Moment Diagram

<u>Condition 2: Fixed Ends</u>

In the elastic range we have the bending moments:

$M_A = -PL/8$, $M_B = -PL/8$ and $M_C = PL/8$
⇨ $P_y = (8/L) \cdot M_y$
⇨ $P_y = (8/0.75) \cdot (23.172)$
⇨ $P_y = 247.17$ N

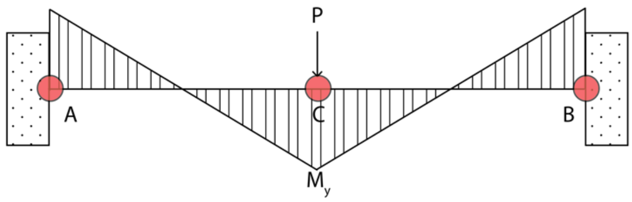


Figure 7 Fixed Ends Moment Diagram

4. <u>Condition 1: Propped Cantilever</u>

L = 750 mm = 0.75 m
Applying the virtual work principle

$W_e = W_i$
⇨ $\sum_i P_i \varphi_i = \sum_j M_{P_j} \varphi_j$
⇨ $\frac{L}{2} \varphi\, P_p = \varphi M_p + 2\varphi M_p$
⇨ $P_p = (6/L) \cdot M_p$
⇨ $P_p = (6/0.75) \cdot (34.758)$
⇨ $P_p = 278.06$ N

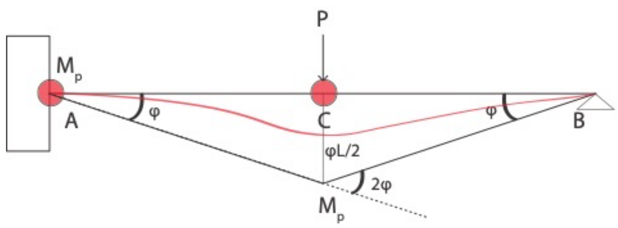


Figure 8 Deformation in Propped Cantilever

<u>Condition 2: Fixed Ends</u>

L = 750 mm = 0.75 m
Applying the virtual work principle

$W_e = W_i$
⇨ $\sum_i P_i \varphi_i = \sum_j M_{P_j} \varphi_j$
⇨ $\frac{L}{2} \varphi\, P_p = \varphi M_p + 2\varphi M_p + \varphi M_p$
⇨ $P_p = (8/L) \cdot M_p$
⇨ $P_p = (8/0.75) \cdot (34.758)$
⇨ $P_p = 370.76$ N

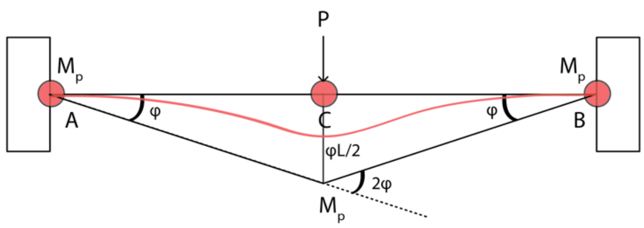


Figure 9 Deformation in Fixed Ends

EXPERIMENTAL DATA 1 : Propped Cantilever

Table 1: Propped cantilever

From the Wizard (*i.e.,* Theoretical results)		
Take values of the:	**M (Nm)**	
Yielding Moment	**23.17**	
Plastic Moment	**34.76**	
Take values of the Loads and Displacements corresponding to the formation of the plastic hinges.	**P (N)**	**δ (mm)**
1st Plastic Hinge	**247.18**	**14.1**
2nd Plastic Hinge	**278.04**	**18.2**
From Experiment (*i.e.,* Experimental results)		
Record the observed values of the Loads and Displacements	**P (N)**	**δ (mm)**
Loading	**0**	**0**
	25	**1.45**
	50	**2.89**
	75	**4.59**
	100	**6.41**
	125	**8.04**
	150	**9.21**
	175	**10.79**
	200	**12.49**
	225	**14.12**
	247.18	**15.76**
	250	**16.3**
	250	**24.86**
	250	**27.76**
	250	**28.12**
	250	**30.35**
Unloading	**200**	**28.32**
	150	**24.94**
	100	**22.1**
	50	**18.41**
	0	**15.18**

POST–EXPERIMENT ANALYSIS 1: Propped Cantilever

1. Comparing theoretical and experimental values for ultimate load capacity

Theoretical Result $P_{y,T}$	Experimental Result $P_{y,E}$
278.04 N	250 N

The theoretical value for the ultimate load capacity obtained from the wizard is the load at which the second hinge is formed, and the experimental value is the maximum load which the beam can take.

The theoretical value is calculated using the value of $f_{y,k}$ the value of which varies with a probability density function. This value varies according to the grade of material manufactured and hence could be the cause for some error in theoretical calculation.

The theoretical result is found to be greater than the experimental result with a percentage experimental error of 11.22%. The sources of error could have mainly resulted due to inaccurate measurements, varying cross-section of beam or wrong positioning of the load.

This shows the importance of applying safety factors in making theoretical calculations for practical building to ensure a certain degree of safety.

2. Difference between theoretical and experimental result

   $P_{y,T} - P_{y,E} = 28.04$ N

   Percentage Experimental Error = $((P_{y,T} - P_{y,E})/P_{y,E}) \times 100$

   ⇨ Percentage Experimental Error = 11.22 %

3. Comparing theoretical and experimental values of displacement

Theoretical Displacement $\delta_{y,T}$	Experimental Displacement $\delta_{y,E}$
18.2 mm	16.3 mm

The experimental value is smaller than the theoretical value. This discrepancy could be due to the incorrect calibration of the supports while conducting the experiment.

4. Ultimate Load Capacity $P_{y,E} = 250$ N

   $M_{y,i} = 5\, P_{y,E}\, L /32$

   ⇨ $M_{y,i} = 29.3$ Pa

   We have $M_y = f_{yk} .(Z_p)$

   ⇨ $f_{y,i} = M_{y,i} / Z_p$

   ⇨ $f_{y,i} = 29.3 / (1.232 \times 10^{-7})$

   ⇨ $f_{y,i} = 23.8 \times 10^{7}$ Pa

⇨ $f_{y,i}$ = 238 MPa

We have f_{yk} = 282 MPa,
⇨ $f_{y,i} < f_{yk}$

Thus, the initial assumed value of the yielding strength is significantly higher than (a difference of 44 MPa) the actual yielding strength of the beam obtained from the experiment.

5. The formation of the first plastic hinge was not clearly observed. There is a vague sense of the hinge forming at some point, but it cannot be certainly observed when the first hinge is formed. However, the formation of the second plastic hinge can more clearly be ascertained as it is where the plastic phase is reached, i.e., when the applied load becomes constant, and the ultimate load capacity is reached. Hence, the second plastic hinge is formed when P = 250 N.

6.

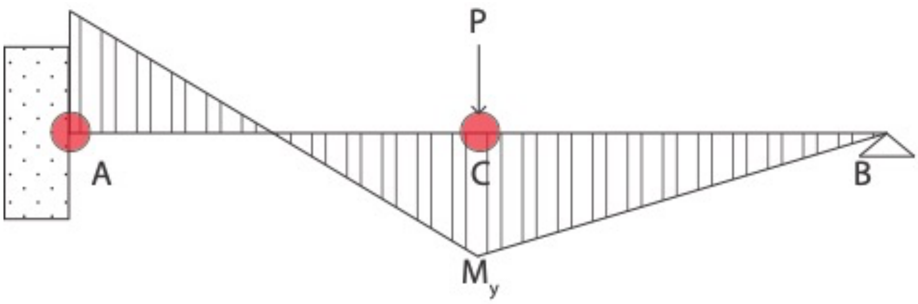


Figure 10 Moment Diagram of Propped Cantilever

The first hinge is formed at the fixed support and the second hinge is formed at mid-span. The maximum bending moment occurs at the fixed support first and a plastic hinge forms first at this position. Due to the support of the prop, however, the beam does not collapse at this stage and requires another plastic hinge before complete collapse occurs. This is formed at the other local position of maximum bending moment at the mid-span position.

This correlates with the theoretical knowledge that when we have more than one hinge forming, the first one forms where the bending moment is the maximum. When this occurs, all fibres of the cross-section have reached plastic stress which gives the plastic moment M_p for that cross section. After this stage, when the load is increased, other cross sections plasticize to form more hinges.

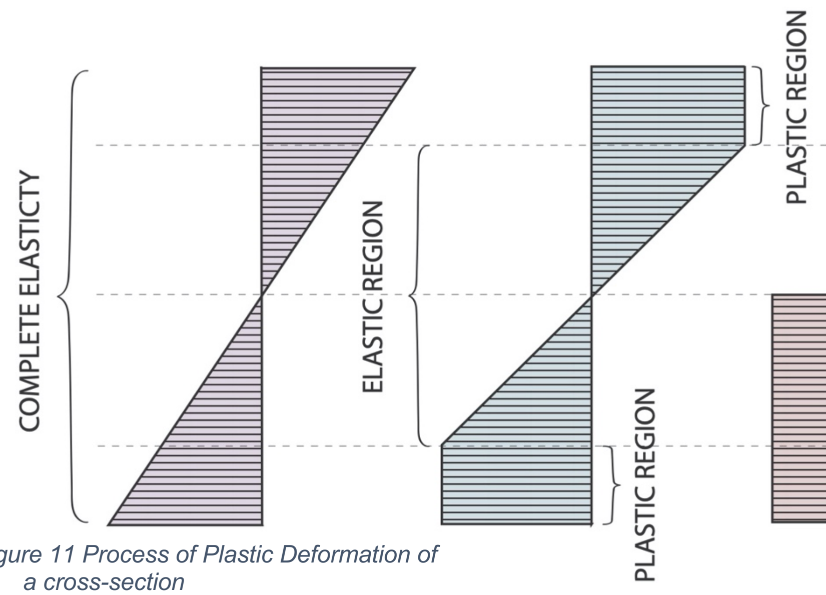
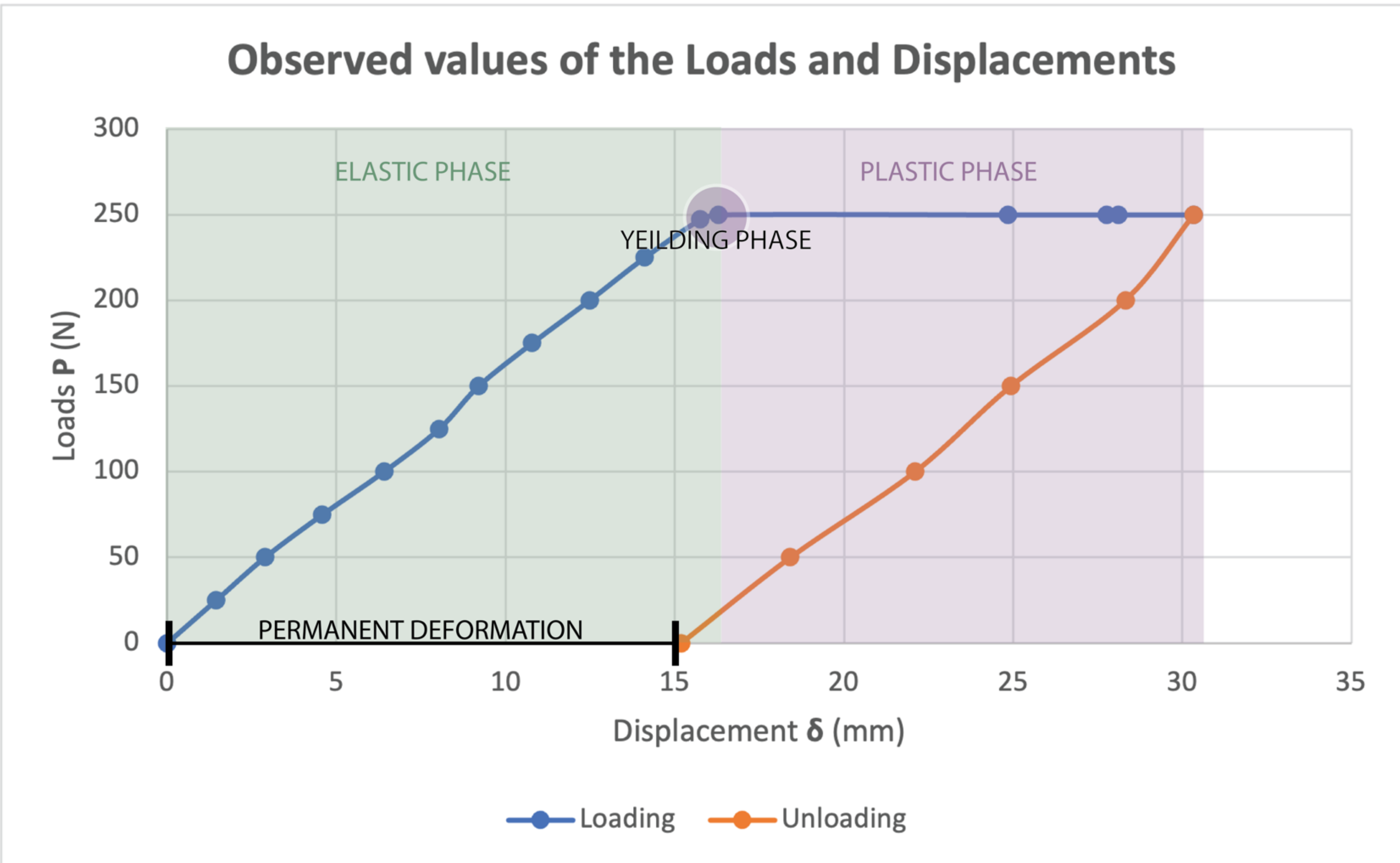


Figure 11 Process of Plastic Deformation of a cross-section

7. The permanent plastic deformation as shown in the force-displacement curve is 15.18 mm



EXPERIMENTAL DATA 2 : Fixed Ends

Table 2: Fixed ends

From the Wizard (*i.e.,* Theoretical results)		
Take values of the:	M (Nm)	
Yielding Moment	23.17	
Plastic Moment	34.76	
Take values of the Loads and Displacements corresponding to the formation of the plastic hinge.	P (N)	δ (mm)
Collapse Load (all 3 plastic hinges formed)	370.77	12.1
From Experiment (*i.e.,* Experimental results)		
Record the observed values of the Loads and Displacements	P (N)	δ (mm)
Loading	0	0
	25	2.91
	50	3.76
	75	4.83
	100	5.89
	125	7.12
	150	7.79
	175	8.77
	200	9.70
	225	10.69
	250	11.61
	275	12.49
	300	13.55
	325	14.28
	350	15.60
	350	17.11
	350	18.20
	350	19.35
	350	26.31
Unloading	300	26.05
	250	25.15
	200	23.76
	150	21.72
	100	19.00
	50	16.19
	0	13.59

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POST–EXPERIMENT ANALYSIS 2: Fixed Ends

1. Comparing theoretical and experimental values for ultimate load capacity

Theoretical Result $P_{y,T}$	Experimental Result $P_{y,E}$
370.77 N	350 N

The theoretical value for the ultimate load capacity obtained from the wizard is the load at which the second hinge is formed, and the experimental value is the maximum load which the beam can take.

The theoretical result is found to be greater than the experimental result with a percentage experimental error of 5.93%. The sources of error could have mainly resulted due to inaccurate measurements, varying cross-section of beam or wrong positioning of the load.

This shows the importance of applying safety factors in making theoretical calculations for practical building to ensure a certain degree of safety.

2. Difference between theoretical and experimental result
   $P_{y,T} - P_{y,E} = 20.77$ N

   Percentage Experimental Error = $((P_{y,T} - P_{y,E})/P_{y,E}) \times 100$
   ⇨ Percentage Experimental Error = 5.93 %

3. Comparing theoretical and experimental values of displacement

Theoretical Displacement $\delta_{y,T}$	Experimental Displacement $\delta_{y,E}$
12.1 mm	15.6 mm

The theoretical displacement is smaller than the experimental displacement, this could be due to improper fixing of the supports during experimentation causing larger displacements.

4. Ultimate Load Capacity $P_{y,E} = 350$ N

   $M_{y,i} = P_{y,E} L /8$
   ⇨ $M_{y,i} = 32.81$ Pa

   We have $M_y = f_{yk} .(Z_p)$

   ⇨ $f_{y,i} = M_{y,i} / Z_p$
   ⇨ $f_{y,i} = 32.81 / (1.232 \times 10^{-7})$
   ⇨ $f_{y,i} = 26.63 \times 10^{7}$ Pa
   ⇨ $f_{y,i} = 266.3$ MPa

We have f_{yk} = 282 MPa,
⇨ $f_{y,i} < f_{yk}$

Thus, the initial assumed value of the yielding strength is significantly higher than (a difference of 15.7 MPa) the actual yielding strength of the beam obtained from the experiment.

5. The formation of all three hinges were observed. However, the exact moment of the formation of the hinges was not clear to observe. It can be estimated that all three hinged were formed during the yielding phase i.e., when plasticity is reached. This would have happened when the ultimate load (350 N) was reached.

6.

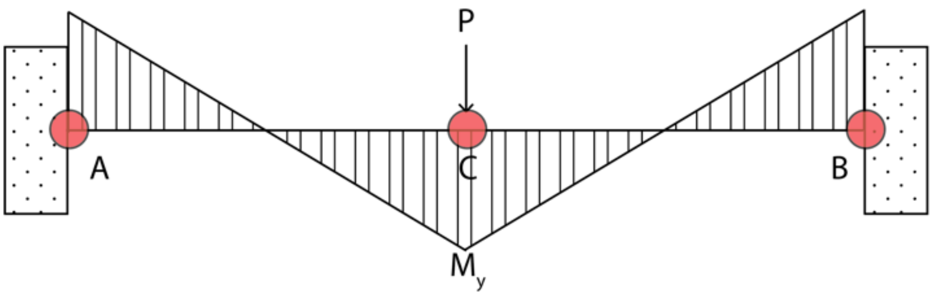


Figure 12 Fixed-Ends Moment Diagram

As is observed from the bending moment diagram and the symmetrical nature of the beam-load-support mechanism, the maximum bending moment is equal and

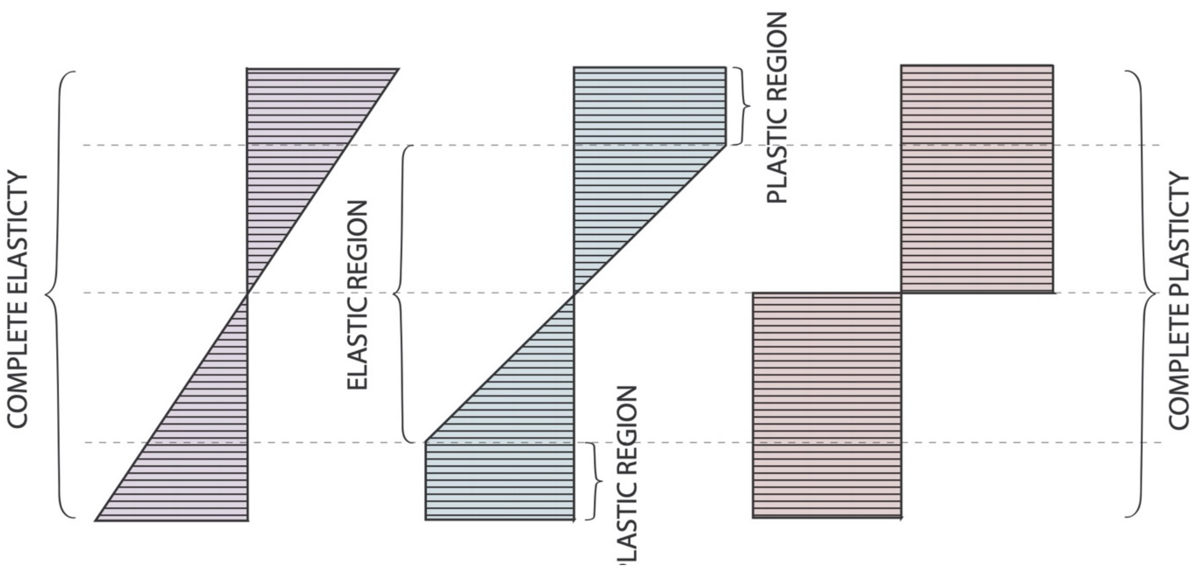
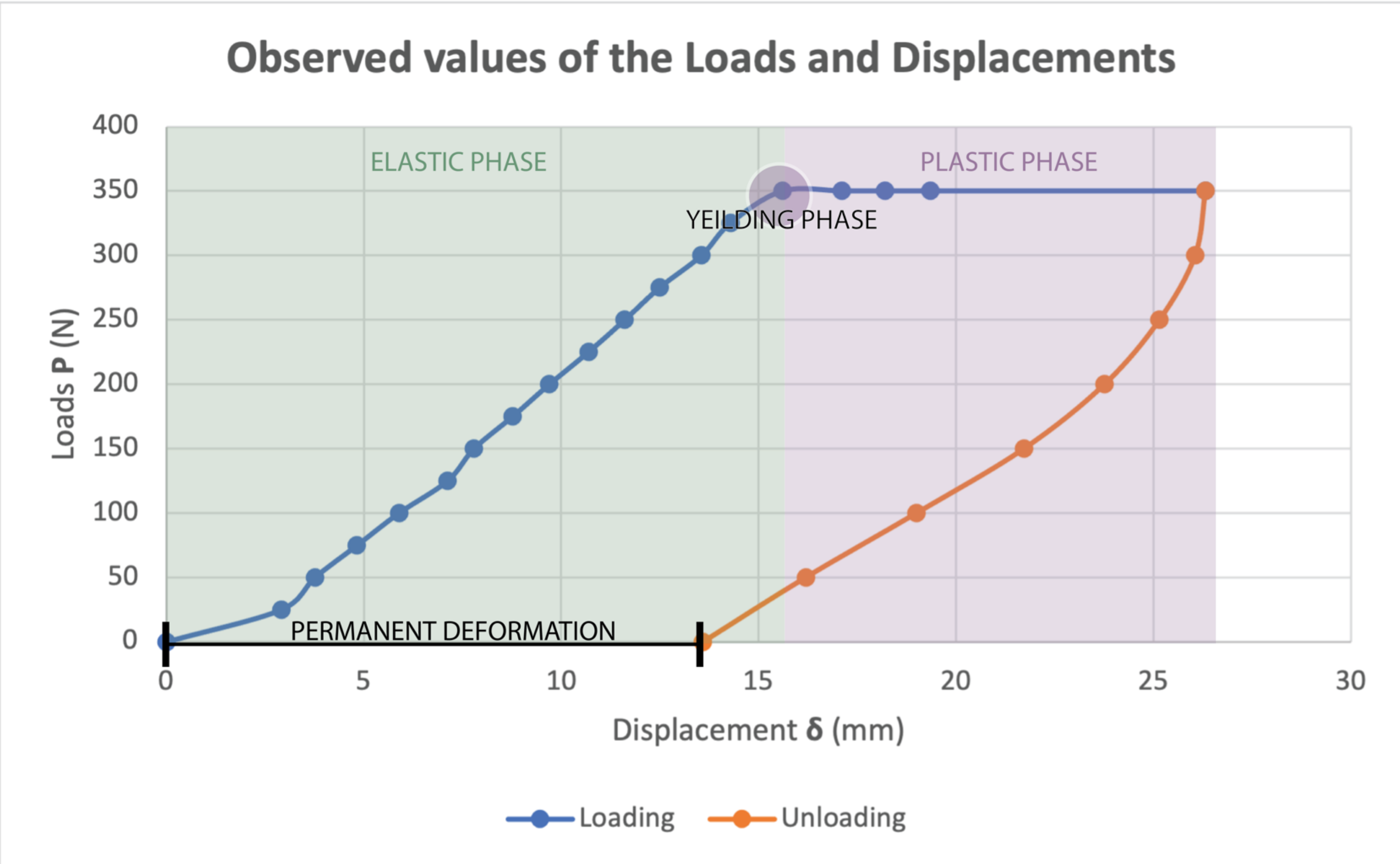


Figure 13 Cross-section showing complete plastic deformation

occurs at points A, B and C. This indicates that all three hinges will be forming at the same time and will form and points A, B and C.

7. The permanent plastic deformation as shown in the force-displacement curve is 13.59 mm.



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